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More Dual String Pairs From Orbifolding

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Abstract

We construct more dual pairs of type II-heterotic strings in four dimensions with $N = 2, 1$ spacetime supersymmetry. On the type II side the construction utilizes the various possible choices of K3 automorphisms with fixed points which transform the holomorphic two-form nontrivially, and rotation plus translation on T^2 . The Calabi-Yau orbifolds so obtained have non-zero Euler numbers, so quantum corrections exist on the type IIA strings. The heterotic string (asymmetric) orbifold duals are found which depend on going to the enhanced symmetry points. Some aspects of the construction are discussed including the role of the singularity and the possibility of going beyond the adiabatic argument. Many of these examples have also orientifold analogs.

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1 Introduction

Since the work of Sen and Harvey and Strominger [1, 2] on the conjectured equivalence of the type IIA string compactified on K3 and the heterotic string compactified on T^4 [3], string-string duality has been a topic of a large amount of recent literature [4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Among the consequences of this conjecture, four dimensional dual string pairs constructed from compactifying further on T^2 and orbifolding [6, 5] are of special interests. On the one hand, it is noted in [5] that due to low energy decoupling of $N = 2$ vector and hypermultiplets, models can be constructed which, by using duality, are capable of producing exact calculations on the quantum moduli space of vector and hypermultiplets on both side. On the other hand, there is indication [14] that the point particle limit of some of the four dimensional dual string pairs constructed in [4] may reproduce the main points of the exact results in the Seiberg-Witten theory [15]. Therefore, it seems likely that a complete understanding of the four dimensional string duality is an important step towards formulating new string theory and deducing its phenomenological consequences.

Despite of the impressive progress, there remain still quite a lot of problems to be solved. For example, we do not know how to deduce the results of [4] from the string-string duality in six dimensions [1, 2] which is better understood than any other analogous conjectures. Initial attempt has been taken in [6] by using adiabatic argument, and by exploring interesting but still mysterious structure of K3-fibration [16, 17, 18] arising in all of the Calabi-Yau hypersurfaces used to compactify type II string duals of [4]. In [13], certain inter-relationship between Kachru-Vafa models is examined from the heterotic side. In [19], it is confirmed that K3-fibration structure is consistent with four dimensional type IIA-heterotic string duality. Other problems include the origin of the enhanced gauge symmetry in the type II dual formulation, which has been actively explored by Aspinwall [20, 21] recently.

In trying to generalize the orbifold construction of [5] to get more dual pairs, instead of using fixed point free K3 automorphisms which are rather rare, one uses automorphisms which act with fixed points on the K3 factor but act freely on the product $K3 \times T^2$ so that the quotient is smooth at least away from the boundary components of the moduli space. In this way, one usually obtains asymmetric orbifolds on the heterotic side, often at the specific point in moduli space with enhanced gauge symmetry. According to [21, 11], the type IIA dual of the enhanced symmetry point corresponds to the degenerate K3 or Calabi-Yau spaces, non-perturbative phenomena such as transition between different phases (topologies) is visible by studying the details of these degenerate points. Thus dual string pairs at the enhanced symmetry point are useful also in understanding phase transition. Perhaps most importantly, non-trivial instanton corrections are present on the heterotic side whose exact form may be deduced from the world sheet instanton corrections on the type IIA side by duality.

In [12, 22, 10, 8] asymmetric orbifolds are constructed in 4 and higher dimensions by utilizing the list of the K3 automorphisms [23] with fixed points. As the above references deal with dual pairs with maximal spacetime supersymmetries, the K3 automorphisms keeping holomorphic two-form invariant are considered. From many phenomenological points of view, string duality in realistic dimensions and with $N = 2$ or less supersymmetry may be more interesting. One way to achieve this, as we will do in this letter, is by using K3 automorphisms which change the holomorphic two-form on K3, but leave invariant a holomorphic three-form on the product $K3 \times T^2$, thus the quotients are smooth Calabi-Yau manifolds on which type IIA string is compactified. The Euler numbers of the Calabi-Yau spaces are different from zero, thus it is probable that there exist worldsheet instanton corrections to the type II moduli space which are equivalent to the spacetime instanton effects under duality. In this letter we will only construct various possible dual pairs and will leave the problems of calculating instanton corrections as well as analyzing phase transitions untouched. It is certainly worthwhile to pursue these questions along the lines of [5, 11].

2 Construction of Dual Pairs

The basic procedure of orbifolding to get dual pairs in 4 dimensions has been outlined in [5, 6, 8]. The strategy is to start from six dimensional type II-Heterotic string duality, and further compactify both side on T^2 . Upon performing various quotient by discrete groups of the K3 automorphism on the type II side, together with rotations and translations of the T^2 , one obtains smooth Calabi-Yau spaces on the type IIA side. Heterotic string duals are obtained by examining how the discrete groups act on the cohomology lattice $H^2(K3 \times T^2, \mathbb{Z})$ which is identified [2] as the heterotic Narain lattice $\Gamma^{22,6}$. For discrete K3 automorphisms of order higher than two, this is achievable by grouping the root lattices of certain Lie algebras embedded in $\Gamma^{19,3}$ according to the fixed point sets of the respective cyclic groups. For the case of Z_2 however, there are fewer choices. We happen to find a Z_2 automorphism which acts on the K3 surface in a way that its lift to the cohomology lattice is obtainable by embedding of the Enriques lattice with finite automorphism group. The heterotic duals then follow from standard identification of moduli spaces of vector and hypermultiplets of the $N = 2$ heterotic string with the invariant and anti-invariant sublattices. As usual, level-matching sometimes requires shift of certain lattice vectors of the heterotic asymmetric orbifolds.

Z_2 Revisited

Our first example comes from modding the $N = 4$ pair of type IIA on $K3 \times T^2$, and heterotic string on $T^4 \times T^2$, by Z_2 .

On the type II side, consider the K3 represented as the quartic hypersurface in CP^3

$$z_1^4 + z_2^4 + z_3^4 + z_4^4 = 0, \quad z_i \in CP^3, \quad (1)$$

the holomorphic (2,0) form is given by

$$\omega_{(2,0)} = \left(\frac{\partial P}{\partial(z_4/z_1)} \right)^{-1} d\left(\frac{z_2}{z_1}\right) \wedge d\left(\frac{z_3}{z_1}\right) = \frac{1}{4z_4^3} (z_1 dz_2 \wedge dz_3 + z_3 dz_1 \wedge dz_2 + z_2 dz_3 \wedge dz_1). \quad (2)$$

P in (2) is the defining polynomial of (1) in homogenous variables. It is easily seen that the following K3 automorphism¹

$$g : (z_1, z_2, z_3, z_4) \longmapsto (-z_1, z_3, z_2, -z_4), \quad (3)$$

transforms $\omega_{(2,0)}$

$$g^* \omega_{(2,0)} = -\omega_{(2,0)}. \quad (4)$$

And for the Z_2 action on $T^2 = S_a^1 \times S_b^1$, we choose the reflection in both factors and a translation on one of the circles, e.g. S_a^1 :

$$g : (x, y) \longmapsto \left(-x + \frac{1}{2}, -y\right). \quad (5)$$

or in the complex coordinate $w = x + iy$ of T^2 :

$$g : w \longmapsto -w + \frac{1}{2}. \quad (6)$$

Note that the Z_2 action in (3) has four fixed points on K3 but the action on the product $K3 \times T^2$ is fixed point free. Without the translation in one of the circles, the total reflection would have four fixed points on the torus T^2 . Since the action on K3 is not free, we have to make the translation in order for the quotient to be smooth. It is worth noting that in the large radius limit $R_a \rightarrow \infty$, one recovers two fixed points on T^2 , and more complicated fixed sets on $K3 \times T^2$.

A variant of the above model for $K3 \times T^2 / Z_2$ can be constructed by considering orientation-reversing involutions on T^2 . There are two possibilities, either by modding out symmetry

$$w \longmapsto \bar{w} + \frac{1}{2} \quad (7)$$

¹It may seem unusual that this automorphism contains a permutation of coordinates. One may however convince oneself that this kind of the K3 symmetry in fact exists in at least a 4 dimensional subspace of the 20 dimensional K3 moduli space, i.e., those arising from polynomial perturbations $\alpha z_1 z_2 z_3 z_4$, $\beta z_2^2 z_3^2$, $\gamma z_2 z_3 z_4^2$, $\delta z_1^2 z_2 z_3$. Given that at the moment the string-string duality admits still "point spectrum", there is no doubt that this symmetry, though existed in a moduli subspace, is worth considering.

or

$$w \longmapsto -\bar{w} + \frac{i}{2} \quad (8)$$

both leading to smooth quotients. In the large radius limit for either of circle a or b , one recovers a singular curve, instead of singular point.

To count the states of this orbifold, we must specify the action of this K3 automorphism on the space of cohomology of $K3 \times T^2$. From the Lefschetz fixed point formula, it follows that

$$4 = 2 - 2 + \text{tr } g|_{H^{1,1}}, \quad (9)$$

where 4 is the number of fixed points, the first 2 on the right comes from invariant (0,0)- and (2,2)-forms, the -2 from the (2,0)-and (0,2)-forms which are odd under (3). Thus we have the eigenvalues of the automorphism g on the 20 (1,1)-forms on K3, $((-1)^8, (1)^{12})$. Combined with the invariant (1,1)-form on T^2 , this gives $h^{1,1} = 13$. For the (2,1)-forms, we have one from combining a holomorphic (2,0) form on K3, $\omega_{(2,0)}$ with a (0,1)-form from the torus which are both Z_2 -odd, and 8 from the odd (1,1)-forms on K3. Therefore $h^{2,1} = 9$. Notice that the resulting Calabi-Yau orbifold has nonvanishing Euler number, in contrast to the case of [5].

In order to find the heterotic dual of this model, we have to first understand how the Z_2 -automorphism acts on the cohomology lattice $H^2(K3 \times T^2, Z)$. Then using the identification of K3 cohomology lattice with the Narain lattice of the toroidal compactification of heterotic string [1, 2, 9], the Z_2 action of (3) is translated into heterotic asymmetric orbifold. Note that the K3 automorphism (3) looks similar to half of an automorphism used to obtain Enriques surface, combined with the one with eight fixed points. Actually K3 can be viewed as the covering space of the Enriques surface. It is thus natural to use finite automorphisms of the Enriques lattice embedded in the K3 lattice to represent the action of (3) on the K3 cohomology lattice. The necessary techniques are explained in [24, 23]. In fact, there exist on Enriques lattice containing rational curves certain root invariants formed by the lattice vectors obeying $q^2 = -2$. Using two of the six root invariants of the Enriques moduli space listed in [23], namely $(D_5 \oplus D_5, \{0\})$ and $(E_7 \oplus A_3, Z_2)$ where the groups $\{0\}, Z_2$ denote the kernels of the homomorphism between Picard groups of K3 and Enriques surface, and the embedding of the Enriques lattice into the K3 lattice:

$$E_8 \oplus H \rightarrow E_8 \oplus E_8 \oplus H \oplus H \oplus H \quad (10)$$

one can determine the Z_2 action on $H^2(K3, Z)$, via the above embedding, as interchanging the two D_5 factors and reversing the sign of the A_3 factor. This can be summarized as follows. One can decompose the cohomology lattice $H^2(K3 \times T^2, Z)$ into even lattices

$$\Gamma^{22,6} = \Gamma^{7,1} \oplus \Gamma^{3,1} \oplus \Gamma_a^{5,1} \oplus \Gamma_b^{5,1} \oplus \Gamma_1^{1,1} \oplus \Gamma_2^{1,1}, \quad (11)$$

where the last two terms correspond to the T^2 . Note that the K3 parts in the decomposition (11) are not self-dual, this is related to the fact that Enriques lattice can not

be embedded into K3 lattice as a direct factor. Let γ^* be the corresponding vector in the lattice Γ^* , then the action is

$$g|\gamma^{7,1}, \gamma^{3,1}, \gamma_a^{5,1}, \gamma_b^{5,1}, \gamma_1^{1,1}, \gamma_2^{1,1}\rangle = |\gamma^{7,1}, -\gamma^{3,1}, \gamma_b^{5,1}, \gamma_a^{5,1}, -\gamma_1^{1,1}, -\gamma_2^{1,1}\rangle, \quad (12)$$

where we assign a twist by null vector in the last direction. It is perhaps worth emphasizing that the above lattice decomposition (11) is not self-dual, thus corresponds not to the generic point in Narain moduli space. This is so, what we obtain is a dual heterotic orbifold at enhanced symmetry point. The action g has ten -1 eigenvalues on the left and four -1 eigenvalues on the right. In particular, the right moving heterotic coordinates corresponding to the holomorphic (2,0)-form of the type II side are projected out, thus the supersymmetry is reduced in this model. It can be shown that the shift of zero point energy on the left is nonzero, thus level-matching requires an additional shift vector be added. This vector $\delta = (p_L, p_R)/2$ must satisfy $p_L^2 - p_R^2 = 3$. The massless spectrum consists of ten hypermultiplets and thirteen vector multiplets, in addition to the gravitational one.

At this point it is interesting to compare with the result of [5]. We obtain the Calabi-Yau on the type IIA side which is not self-mirror, i.e. $\chi \neq 0$. This must mean that there exist non-perturbative instanton corrections on the type II side which maps to the spacetime instanton corrections on the heterotic side. Further analysis is needed in order to use the (first quantized) mirror map to calculate the instanton correction on the heterotic side, this could provide a nontrivial test of the 2nd quantized mirror symmetry proposed in [5].

Other Cyclic Groups

The analysis for the other cyclic groups can be made in the same fashion. For $G = Z_p$, $p = 3, 5, 7$, the action of G on the K3 is by multiplying group elements $\exp(2ki\pi/p)$, $0 \leq k < p$ to the appropriate complex variables z_i , $i = 1, \dots, 4$ of some weighted CP^3 in which the K3 is realized. And on the torus T^2 , the G action is such that rotation on one of the circles accompanies the translation in order to compensate for the nontrivial transformation of the holomorphic form on K3 while retaining fixed point free action on the product $K3 \times T^2$. For example, take $G = Z_3$, one notices that the K3 automorphisms

$$g_1 : (z_1, z_2, z_3, z_4) \mapsto (e^{2i\pi/3} z_1, e^{2i\pi/3} z_2, z_3, z_4), \quad (13)$$

$$g_2 : (z_1, z_2, z_3, z_4) \mapsto (z_1, z_2, e^{2i\pi/3} z_3, e^{2i\pi/3} z_4) \quad (14)$$

of Z_3 transform the holomorphic two-form as

$$g_1^* \omega_{(2,0)} = e^{2i\pi/3} \omega_{(2,0)}, \quad (15)$$

$$g_2^* \omega_{(2,0)} = e^{-2i\pi/3} \omega_{(2,0)} \quad (16)$$

which can be rendered invariant if one tensors it with dw from the T^2 with nontrivial Z_3 transformations. The number of fixed points under either g_1 or g_2 is nine; it is six under both. Counting of states can be carried out similarly. For example, from the untwisted sector we have two invariant (1,1)-forms, one of them is the Kähler form. There are eighteen anti self-dual (1,1)-forms which are cyclically interchanged by g_1, g_2 , thus by forming linear combinations six out of them are also Z_3 -invariant. Unlike the Z_2 case, now a Z_3 transformation does not leave the (1,1)-form from T^2 invariant (the same is true for other higher order cyclic groups). So we obtain $h^{11} = 8$. The invariant (2,1)-forms can be counted as follow. There is one coming from tensoring $\omega_{(2,0)} \otimes d\bar{w}$ on $K3 \times T^2$. From the twisted sector, we get six (1,1)-forms which when tensored by dw from T^2 are Z_3 -invariant. Since there are two twisted sectors, the final result is $h^{21} = 13$.

The heterotic dual of this orbifold can be realized as an asymmetric orbifold with enhanced gauge symmetry similar to that of [12, 10]. We consider the decomposition of the Narain lattice as follows:

$$\Gamma^{22,6} = E_8 \oplus E_8 \oplus \Gamma_1^{2,2} \oplus \Gamma_2^{2,2} \oplus \Gamma_3^{2,2} \quad (17)$$

which contains the enhanced symmetry of type $(D_1^6)^3 \times D_1^2 \times D_1^4$ embedded in the 24 dimensional Niemeier lattice D_6^4 . It is then not difficult to specify the Z_3 -action on (17): The three D_1^6 factors are cyclically interchanged under Z_3 . Since the right-moving coordinates corresponding to the self-dual two-forms on the type IIA side transform nontrivially, supersymmetry is reduced. Level-matching requires shifting the lattice $\Gamma_2^{2,2}$ by a vector $\delta^2 = \frac{1}{3}$, in addition to the the null vector shift in the third factor $\Gamma_3^{2,2}$.

For groups Z_5, Z_7 , one can go on to find the Calabi-Yau orbifolds, but now since there would be no room for putting shift in the lattice, level-matching rules out the existence of corresponding heterotic duals. Coincidentally, the Calabi-Yau manifolds obtained in these examples have the same Hodge numbers as those obtained from the examples of Z_6 and Z_8 , respectively. Thus there is no loss as far as getting heterotic-type IIA dual pairs is concerned.

For $G = Z_m$, $m = 4, 6, 8$, to construct Calabi-Yau orbifolds, care must be taken since the actions by the subgroups Z_2, Z_3 may be different from that of the higher order elements. For example, Z_4 has two elements of order four, one element of order two, acting by rotation by $\pi/2$ and reflection, respectively, on the complex coordinates of the K3. For the K3 realized as in (1), one would not obtain Calabi-Yau orbifold by pair-wise action on the coordinates since the order two element always acts preserving holomorphic two-form $\omega_{(2,0)}$. Likewise, the choice of action of the Z_2 -subgroup as in (3) would not be appropriate since then one is actually modding by the semi-direct product group $Z_4 \times S_4$ which could not be translated into conventional dual heterotic orbifold. Nevertheless, if one chooses the Z_4 -action such that the order four element acts by multiplying $\omega_{(2,0)}$ by $\pm i$, the order-two element maps it nontrivially as well.

Note that for this Z_4 example, there are eight fixed points for each of the order-four elements, and 12 for the order-two element. Only 4 of them are common fixed points. Standard counting gives the Hodge numbers of this Z_4 orbifold as $h^{1,1} = 6$, $h^{2,1} = 15$. And similarly we have $h^{1,1} = 4$, $h^{2,1} = 17$ for Z_6 , and $h^{1,1} = 2$, $h^{2,1} = 19$ for Z_8 . All of these type II Calabi-Yau orbifolds have (asymmetric) heterotic orbifold duals near the enhanced symmetry point. We summarize the result in Table 1.

Product Groups

Now we consider the case of $K3 \times T^2/G$, where $G = Z_2 \times Z_2$, $Z_2 \times Z_4$, $Z_3 \times Z_3$, $Z_4 \times Z_4$, $Z_2 \times Z_6$. These groups have a number of subgroups which fall into the classes in the previous examples. In general, one obtains $N = 1$ spacetime supersymmetry by suitably choosing the G -action on the right-movers. It is also possible to obtain $N = 2$ dual pairs using the methods of [12]. Here we only consider orbifold with $N = 1$ spacetime supersymmetry, thus we require that holomorphic two-form on K3 is transformed by elements of both factors.

Let us consider the modding by group $Z_2 \times Z_2$ which contains three Z_2 subgroups. Denoting their elements as g_1, g_2 and g_3 , respectively, we take the action of g_1 as in (3) and that of g_2 as the "conjugate" of g_1 in such a way that the third $g_3 = g_2 g_1$ acts by even permutations of two pairs of the coordinates, z_i , $i = 1, \dots, 4$, and without reflections. There are two subtle points worth noting. Firstly, one cannot at the same time choose all three of g_i acting on K3 as in (3) for reason that S_4 has no order two subgroup needed to make the actions of g_1 and g_2 commute. Secondly, the action of the product $g_3 = g_2 g_1$ necessarily preserves the holomorphic two-form on K3, as is easily checked. This is consistent with the transformation of $Z_2 \times Z_2$ on T^2 , where the product of the two commuting transformations of the type (6) preserves the one-form dw . For the above choice of the $Z_2 \times Z_2$ action on $K3 \times T^2$, it is not difficult to see that there are four fixed points for each of g_1, g_2 , and eight for g_3 . Examining this $Z_2 \times Z_2$ action on $H^2(K3)$, one obtains the following spectrum of the type IIA Calabi-Yau orbifold. There are five (1,1)-forms surviving the $Z_2 \times Z_2$ projection. The other sixteen anti-self-

G	$\#Fix(K3)$	$n_v = h^{1,1}$	$n_h = h^{2,1} + 1$	shift δ^2
Z_2	4	13	10	3/4
Z_3	6	8	14	1/3
Z_4	4	6	16	1/4
Z_6	2	4	18	5/4
Z_8	2	2	20	1/8

Table 1: Type II Calabi-Yau orbifolds whose heterotic duals can be found. The second column indicates the number of fixed points on the K3. The last column shows the shift vectors required to satisfy the level-matching condition.

dual (1,1)-forms are generically interchanged. Especially spacetime supersymmetry is broken to $N = 1$. Since the Kähler mode is left invariant in this case, of the five $N = 2$ vector multiplets which are even under $Z_2 \times Z_2$ each contributes an additional chiral multiplet after truncation to $N = 1$. Combined with sixteen truncated hypermultiplets and three corresponding to the T^2 moduli S, T, U , this yields 24 chiral multiplets. We have therefore obtained the field content of a $N = 1$ supergravity coupled to five $N = 1$ vector supermultiplets and 24 massless chiral multiplets.

Now we map this to the $N = 1$ heterotic dual. We know the $Z_2 \times Z_2$ action on the cohomology lattice of $K3 \times T^2$, which is then translated into corresponding action on the Narain lattice from the heterotic side. The eigenvalues of three Z_2 elements are shown below (left and right components are separated by ';' and repeated eigenvalues are denoted by superscripts)

$$g_1 : (1^7, (-1)^3, 1^5, (-1)^5, (-1)^2; 1, -1, 1, -1, (-1)^2) \quad (18)$$

$$g_2 : (1^5, (-1)^5, 1^7, (-1)^3, (-1)^2; 1, -1, 1, -1, (-1)^2) \quad (19)$$

$$g_3 : (1^2, (-1)^8, 1^2, (-1)^8, 1^2; 1, -1, 1, -1, 1^2). \quad (20)$$

Note that the product action g_3 satisfies level-matching without additional shifts. The massless spectrum includes five vector multiplets coming from the untwisted sector. Generically there are no massless states in the twisted sectors. The invariant projection of the states $\alpha_{-1}^I |0\rangle_L \otimes |i\rangle_R$, $I = 1, \dots, 22, i = 1, \dots, 6$ gives the 24 scalar multiplets including gravitational state. Note that dilaton is now in a chiral multiplet. At the enhanced symmetry point the low energy gauge group is of the form $SO(4) \times SO(4) \times U(1)$ with the last factor coming from the invariant $N = 2$ vector multiplet containing dilaton. At the self-dual radius of one of the circle in T^2 , one gets $SO(4) \times SO(4) \times SU(2)$. The ability of having an $SU(2)$ point independent of the enhanced symmetry point in the bulk of moduli space may be phenomenologically interesting as one can study low energy effective theory with different field content and gauge groups by Higgsing along different directions. Examining the worldsheet currents at the enhanced symmetry point tells us that the gauge group is at Kac-Moody level four.

Other $N = 1$ dual pairs from orbifolding by the product groups G are similarly worked out with the results summarized in Table 2.

3 Comments and Discussions

All the dual pairs in the preceding section are obtained via standard orbifold techniques, they appear to parallel the existing dictionary of the type II-heterotic string duality in four dimensions. Some special features of this construction are worth emphasizing.

Firstly, the singularities at the large radius limit in our examples seemed to invalidate the adiabatic argument of [6]. Indeed, fiber-wise application of the six dimensional string duality requires smooth action of the orbifolding group on the base, for sufficiently large radius. The presence of quotient singularities at large radius limit certainly says that there are subtleties in applying string duality near the boundary of the moduli space. In four dimensional heterotic string, dilaton comes from the modulus (area) of the two-torus, thus large radius limit corresponds to weak coupling of the heterotic string. Appearance of enhanced symmetry in the weakly coupled limit has been recently observed [21] at least for the perturbatively visible parts of the group [19]. The question then is, can we prove that taking the weak-coupling limit commutes with going to the boundary corresponding to the enhanced gauge symmetry. An affirmative answer clearly saves the adiabatic arguments from suffering from singularity, at least those appearing in the large radius limit. Recently, enhanced gauge symmetry points are examined from the D-string point of view [25], among other advantages, this enables more geometric description of the degeneration within Calabi-Yau threefolds. One of the new results is that, instead of shrinking down to zero size the rational curves, the process is better understood as vanishing of 3-cycles [26], $S_{ij}^2 \times (A, B)$ where S_{ij}^2 comes from the singular locus of K3, and the A, B are either of the circles of T^2 . The cycles containing A, B are never mixed. It is clear from this picture that one can make the the two processes commute, thus avoiding contradiction with adiabatic argument.

Another point of interests in our construction is the use of the large radius limit singularity in realizing the extremal transition [11]. The phase transition usually occurs when one blows down certain rational curves followed by blowing-ups. The ability to blow down curves nontrivially is restricted by something nontrivial a finite distance away from the boundary of the moduli space, i.e., the singular loci resulting from blown-down curves must be identified by the quotienting group in question. In our examples, the quotient singularities are naturally interpreted as fixed points of the Weyl reflection of the root lattices of the a, d, e Lie algebras, thus their resolution will provide more possibilities of phase transitions, perhaps leading to more realistic pairs.

G	vector	chiral	enhanced points	K-M level
$Z_2 \times Z_2$	5	24	$SO(4) \times SO(4) \times SU(2)$	4
$Z_2 \times Z_4$	3	21	$SU(2) \times SO(4)$	8
$Z_2 \times Z_6$	2	16	$SU(2)^2$	12
$Z_3 \times Z_3$	4	19	$SO(8)$	9
$Z_4 \times Z_4$	2	17	$SO(4)$	16

Table 2: $N = 1$ heterotic duals to the type IIA on $K3 \times T^2/G$. Indicated here are also possible low energy gauge groups which appear at the enhanced symmetry points, and their Kac-Moody levels.

It is also worth mentioning that most of the examples in this letter can be generalized to the orientifold construction, using e.g, the automorphisms on T^2 given by (7), (8), together with reversion of worldsheet helicity. But as there are no new insights in the construction, we will not report the result here.

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